Worst-Case Value-at-Risk of Derivative Portfolios

Steve Zymler  Berç Rustem  Daniel Kuhn

Department of Computing
Imperial College London

Thalesians Seminar Series, November 2009
“Since the meltdown of some of Wall Street’s biggest names, there has been a great deal of talk, even in quant circles, that this widespread institutional reliance on \textit{VaR was a terrible mistake}. At the very least, the risks that \textit{VaR measured did not include the biggest risk of all: the possibility of a financial meltdown}.”

“To me, VaR is charlatanism because it tries to estimate something that is not scientifically possible to estimate, namely the risks of rare events. It gives people misleading precision that could lead to the buildup of positions by hedgers. It lulls people to sleep. All that because there are financial stakes involved.”

– Nassim Taleb

Outline

1. Introduction to Convex Optimization
2. Portfolio Optimization and Value-at-Risk
3. Worst-Case Value-at-Risk and Robust Optimization
4. Worst-Case Value-at-Risk of Derivative Portfolios
5. Numerical Results: Beating the S&P 500
6. Conclusions
Consider the general optimization problem:

\[
\begin{align*}
\text{minimize} & \quad f_0(x) \\
\text{subject to} & \quad f_i(x) \leq 0, \quad i = 1, \ldots, m \\
& \quad x \in \mathbb{R}^n
\end{align*}
\]

where:

- \( f_0(x) \) is the objective function
- \( f_i(x) \leq 0 \) are the constraints
- \( x \in \mathbb{R}^n \) are the decision variables

If \( f_0, f_1, \ldots, f_m \) are convex in \( x \) then:

- (P) is polynomial time solvable (tractable)

If not then:

- (P) is a global optimization problem \( \rightarrow \) NP-hard (intractable)
The class of Conic Programs is comprised of:

- **Linear Programs (LP):**
  \[
  \begin{aligned}
  &\text{minimize} & & c^T x \\
  &\text{subject to} & & Ax \leq b
  \end{aligned}
  \]

- **Second-Order Cone Programs (SOCP):**
  \[
  \begin{aligned}
  &\text{minimize} & & c^T x \\
  &\text{subject to} & & \|A_ix + b_i\|_2 \leq f_i^T x + d_i, \quad i = 1, \ldots, m
  \end{aligned}
  \]

- **Semidefinite Programs (SDP):**
  \[
  \begin{aligned}
  &\text{minimize} & & c^T x \\
  &\text{subject to} & & F_0 + \sum_{i=1}^{n} F_ix_i \succeq 0
  \end{aligned}
  \]

- Can all be solved using very efficient software packages
Second-Order Cone or Ice Cream Cone

\( \{(x, y, z) : \sqrt{x^2 + y^2} \leq z\} \)
Positive Semidefinite Cone

\( \left\{ \begin{bmatrix} x & y \\ y & z \end{bmatrix} \in S^2 : \begin{bmatrix} x & y \\ y & z \end{bmatrix} \succeq 0 \right\} \)
For every Primal Problem a Dual Problem can be constructed:

- **Primal Problem:**
  
  minimize \( c^T x \)  
  subject to \( Ax \leq b, \quad x \geq 0 \) \( (P) \)

- **Dual Problem:**
  
  maximize \( b^T y \)  
  subject to \( A^T y \geq c, \quad y \geq 0 \) \( (D) \)

- If \((P)\) is feasible, i.e.:

  \[ \exists \hat{x} \in \mathbb{R}^n \text{ such that } A\hat{x} \leq b, \quad \hat{x} \geq 0 \]

then

\[ \text{Opt}(P) = \text{Opt}(D) \quad (\text{Strong Duality}) \]
Consider a market consisting of \( m \) assets.

**Optimal Asset Allocation Problem**

Choose the weights vector \( \mathbf{w} \in \mathbb{R}^m \) to make the portfolio return high, whilst keeping the associated risk \( \rho(\mathbf{w}) \) low.

**Portfolio optimization problem:**

\[
\begin{align*}
\text{minimize} & \quad \rho(\mathbf{w}) \\
\text{subject to} & \quad \mathbf{w} \in \mathcal{W}.
\end{align*}
\]

**Popular risk measures \( \rho \):**

- **Variance →** Markowitz model (1952)
- **Value-at-Risk →** Focus of this talk
Let $\tilde{r}$ denote the random returns of the $m$ assets.

The portfolio return is therefore $w^T \tilde{r}$.

**Value-at-Risk (VaR)**

The minimal level $\gamma \in \mathbb{R}$ such that the probability of $-w^T \tilde{r}$ exceeding $\gamma$ is smaller than $\epsilon$.

$$\text{VaR}_\epsilon(w) = \min \left\{ \gamma : \mathbb{P} \left\{ \gamma \leq -w^T \tilde{r} \right\} \leq \epsilon \right\}$$
Theoretical and Practical Problems of VaR

- VaR lacks some desirable theoretical properties:
  - Not a coherent risk measure.
  - Needs precise knowledge of the distribution of $\tilde{r}$.
  - Non-convex function of $w$ → VaR minimization intractable.

- To optimize VaR: resort to VaR approximations.

- Example: assume $\tilde{r} \sim \mathcal{N}(\mu_r, \Sigma_r)$, then

  $$\text{VaR}_\epsilon(w) = -\mu_r^T w - \Phi^{-1}(\epsilon) \sqrt{w^T \Sigma r w},$$

- Normality assumption unrealistic (CLT not applicable) → may underestimate the actual VaR!

- Very difficult to know “true” distribution: model risk
Worst-Case Value-at-Risk

- Only know means $\mu_{\tilde{r}}$ and covariance matrix $\Sigma_{\tilde{r}} \succ 0$ of $\tilde{r}$.
- Let $\mathcal{P}_r$ be the set of all distributions of $\tilde{r}$ with mean $\mu_{\tilde{r}}$ and covariance matrix $\Sigma_{\tilde{r}}$.

**Worst-Case Value-at-Risk (WCVaR)**

$$\text{WCVaR}_{\epsilon}(w) = \min \left\{ \gamma : \mathbb{P}\left\{ \gamma \leq -w^T\tilde{r} \right\} \leq \epsilon \quad \forall \mathbb{P} \in \mathcal{P}_r \right\}$$

- WCVaR is immunized against uncertainty in $\mathbb{P}$: distributionally robust.
- Unless the most pessimistic distribution in $\mathcal{P}_r$ is the true distribution, actual VaR will be lower than WCVaR.
Worst-Case Value-at-Risk

- Only know means $\mu_r$ and covariance matrix $\Sigma_r \succ 0$ of $\tilde{r}$.

- Let $\mathcal{P}_r$ be the set of all distributions of $\tilde{r}$ with mean $\mu_r$ and covariance matrix $\Sigma_r$.

Worst-Case Value-at-Risk (WCVaR)

$$WCVaR_\epsilon(w) = \min \left\{ \gamma : \max_{\mathcal{P} \in \mathcal{P}_r} \mathbb{P}\left\{ \gamma \leq -w^T\tilde{r} \right\} \leq \epsilon \right\}$$

- WCVaR is immunized against uncertainty in $\mathbb{P}$: distributionally robust.

- Unless the most pessimistic distribution in $\mathcal{P}_r$ is the true distribution, actual VaR will be lower than WCVaR.
El Ghaoui et al. have shown that

\[
\text{WCVaR}_\epsilon(w) = -\mu^T w + \kappa(\epsilon)\sqrt{w^T \Sigma w},
\]

where \( \kappa(\epsilon) = \sqrt{(1 - \epsilon)/\epsilon} \).

Key idea behind proof:

For any fixed \( \gamma \in \mathbb{R} \) and \( w \in \mathcal{W} \), define \( S \) as

\[
S = \{ r \in \mathbb{R}^m : \gamma \leq -w^T r \}
\]

Then we have

\[
\max_{\mathbb{P} \in \mathcal{P}_r} \mathbb{P} \{ \gamma \leq -w^T \tilde{r} \} = \max_{\mathbb{P} \in \mathcal{P}_r} \mathbb{P} \{ \tilde{r} \in S \}
\]

We also define \( \mathbb{I}_S \) as

\[
\mathbb{I}_S(r) = \begin{cases} 
1 & \text{if } r \in S \\
0 & \text{otherwise}
\end{cases}
\]
Resolving Integration via Dualization

We can write \( \pi = \max_{P \in \mathcal{P}_r} \mathbb{P} \{ \tilde{r} \in S \} \) as

\[
\pi = \max_{\mu \in \mathcal{M}_+} \int_{\mathbb{R}^m} \mathbb{I}_S(r) \mu(dr)
\]

s. t. \( \int_{\mathbb{R}^m} \mu(dr) = 1 \)

\[
\int_{\mathbb{R}^m} \mu(dr) = \mu_r
\]

\[
\int_{\mathbb{R}^m} rr^T \mu(dr) = \Sigma_r + \mu_r \mu_r^T,
\]

Key idea: construct the dual!

\[
\begin{align*}
\min & \quad y_0 + y^T \mu_r + \langle Y, \Sigma_r + \mu_r \mu_r^T \rangle \\
\text{s. t.} & \quad y_0 \in \mathbb{R}, \quad y \in \mathbb{R}^m, \quad Y \in \mathbb{S}^m
\end{align*}
\]

It can be shown that: \( \text{Opt}(P) = \text{Opt}(D) \)
It can be shown that:

$$\text{WCVaR}_\epsilon(\mathbf{w}) = \max_{\mathbf{r} \in \mathcal{U}_\epsilon} -\mathbf{w}^T \mathbf{r},$$

where the ellipsoidal uncertainty set $\mathcal{U}_\epsilon$ is defined as

$$\mathcal{U}_\epsilon = \left\{ \mathbf{r} \in \mathbb{R}^m : (\mathbf{r} - \mu_r)^T \Sigma_r^{-1} (\mathbf{r} - \mu_r) \leq \kappa(\epsilon)^2 \right\}$$

Therefore,

$$\min_{\mathbf{w} \in \mathcal{W}} \text{WCVaR}_\epsilon(\mathbf{w}) \equiv \min_{\mathbf{w} \in \mathcal{W}} \max_{\mathbf{r} \in \mathcal{U}_\epsilon} -\mathbf{w}^T \mathbf{r}$$

Can be interpreted as a game versus nature which acts as a purely adverserial player.
Robust Optimization Perspective on WCVaR

Zymler, Rustem and Kuhn

Worst-Case Value-at-Risk of Derivative Portfolios
Extensions

- Incorporate Moment Uncertainty:
  - Mean Uncertainty:
    \[ \mathcal{U}_\mu = \{ \mu : \underline{\mu} \leq \mu \leq \overline{\mu} \} \]
  - Covariance Matrix Uncertainty:
    \[ \mathcal{U}_\Sigma = \{ \Sigma \succeq 0 : \Sigma \preceq \Sigma \preceq \overline{\Sigma} \} \]
  - Inject moment uncertainty into ellipsoid:
    \[ \mathcal{U}_\epsilon = \{ \mathbf{r} \in \mathbb{R}^m : \exists \Sigma_r \in \mathcal{U}_\Sigma, \ (\mathbf{r} - \mu_r)^T \Sigma_r^{-1} (\mathbf{r} - \mu_r) \leq \kappa(\epsilon)^2 \} \]

- Incorporate Support Information:
  - Let \( \mathcal{B} \) be the smallest set such that \( \mathbb{P} \{ \tilde{\mathbf{r}} \in \mathcal{B} \} = 1 \)
  - Can be used to make WCVaR less conservative:
    \[ \mathcal{U}_\epsilon = \{ \mathbf{r} \in \mathcal{B} : (\mathbf{r} - \mu_r)^T \Sigma_r^{-1} (\mathbf{r} - \mu_r) \leq \kappa(\epsilon)^2 \} \]
Assume that the market consists of:

- $n \leq m$ basic assets with returns $\tilde{\xi}$, and
- $m - n$ derivatives with returns $\tilde{\eta}$.
- $\tilde{\xi}$ are only risk factors.

We partition asset returns as $\tilde{r} = (\tilde{\xi}, \tilde{\eta})$.

Derivative returns $\tilde{\eta}$ are uniquely determined by basic asset returns $\tilde{\xi}$. There exists $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ with $\tilde{r} = f(\tilde{\xi})$.

$f$ is highly non-linear and can be inferred from:

- Contractual specifications (option payoffs)
- Derivative pricing models
WCVaR is applicable but not suitable for portfolios containing derivatives:

- Moments of $\tilde{\eta}$ are difficult to estimate accurately.
- Disregards perfect dependencies between $\tilde{\eta}$ and $\tilde{\xi}$.

WCVaR severly overestimates the actual VaR, because:

- $\Sigma_r$ only accounts for linear dependencies
- $\mathcal{U}_\epsilon$ is symmetric but derivative returns are skewed
Generalized Worst-Case VaR Framework

We develop two new Worst-Case VaR models that:
- Use first- and second-order moments of $\tilde{\xi}$ but not $\tilde{\eta}$.
- Incorporate the non-linear dependencies $f$

**Generalized Worst-Case VaR**

Let $\mathcal{P}$ denote set of all distributions of $\tilde{\xi}$ with mean $\mu$ and covariance matrix $\Sigma$.

$$\min \left\{ \gamma : \max_{\mathcal{P} \in \mathcal{P}} \mathbb{P} \left\{ \gamma \leq -w^T f(\tilde{\xi}) \right\} \leq \epsilon \right\}$$

When $f(\tilde{\xi})$ is:
- convex polyhedral $\rightarrow$ Worst-Case Polyhedral VaR ($\text{SOCP}$)
- nonconvex quadratic $\rightarrow$ Worst-Case Quadratic VaR ($\text{SDP}$)
Assume that the $m - n$ derivatives are European put/call options maturing at the end of the investment horizon $T$.

Basic asset returns: $\tilde{r}_j = f_j(\tilde{\xi}) = \tilde{\xi}_j$ for $j = 1, \ldots, n$.

Assume option $j$ is a call with strike $k_j$ and premium $c_j$ on basic asset $i$ with initial price $s_i$, then $\tilde{r}_j$ is

$$f_j(\tilde{\xi}) = \frac{1}{c_j} \max \left\{ 0, s_i(1 + \tilde{\xi}_i) - k_j \right\} - 1$$

$$= \max \left\{ -1, a_j + b_j \tilde{\xi}_i - 1 \right\}, \text{ where } a_j = \frac{s_i - k_j}{c_j}, \quad b_j = \frac{s_i}{c_j}.$$

Likewise, if option $j$ is a put with premium $p_j$, then $\tilde{r}_j$ is

$$f_j(\tilde{\xi}) = \max \left\{ -1, a_j + b_j \tilde{\xi}_i - 1 \right\}, \text{ where } a_j = \frac{k_j - s_i}{p_j}, \quad b_j = -\frac{s_i}{p_j}.$$
In compact notation, we can write $\tilde{r}$ as

$$\tilde{r} = f(\tilde{\xi}) = \max \left\{ -e, a + B\tilde{\xi} - e \right\}.$$ 

Partition weights vector as $w = (w^\xi, w^\eta)$.

No derivative short-sales: $w \in \mathcal{W} \implies w^\eta \geq 0$.

Portfolio return of $w \in \mathcal{W}$ can be expressed as

$$w^T\tilde{r} = w^T f(\tilde{\xi})$$

$$= (w^\xi)^T \tilde{\xi} + (w^\eta)^T \max \left\{ -e, a + B\tilde{\xi} - e \right\}.$$
Use the piecewise linear portfolio model:

\[ w^T f(\tilde{\xi}) = (w^\xi)^T \tilde{\xi} + (w^\eta)^T \max \left\{ -e, a + B\tilde{\xi} - e \right\}. \]

Worst-Case Polyhedral VaR (WCPVaR)

For any \( w \in \mathcal{W} \), we define \( \text{WCPVaR}_\epsilon(w) \) as

\[
\text{WCPVaR}_\epsilon(w) = \min \left\{ \gamma : \max_{P \in \mathcal{P}} \left\{ \gamma \leq -w^T f(\tilde{\xi}) \right\} \leq \epsilon \right\}.
\]
Worst-Case Polyhedral VaR: Convex Reformulations

**Theorem: SDP Reformulation of WCPVaR**

WCPVaR of \( w \) can be computed as an SDP:

\[
\text{WCPVaR}_\epsilon(w) = \min \gamma \\
\text{s.t.} \quad M \in S^{n+1}, \quad y \in \mathbb{R}^{m-n}, \quad \tau \in \mathbb{R}, \quad \gamma \in \mathbb{R} \\
\langle \Omega, M \rangle \leq \tau \epsilon, \quad M \succeq 0, \quad \tau \geq 0, \quad 0 \leq y \leq w^n \\
M + \begin{bmatrix} 0 & w^\xi + B^T y \\ (w^\xi + B^T y)^T & -\tau + 2(\gamma + y^T a - e^T w^n) \end{bmatrix} \succeq 0
\]

Where we use the second-order moment matrix \( \Omega \):

\[
\Omega = \begin{bmatrix} \Sigma + \mu \mu^T & \mu \\ \mu^T & 1 \end{bmatrix}
\]
Theorem: SOCP Reformulation of WCPVaR

WCPVaR of $w$ can be computed as an SOCP:

$$
\text{WCPVaR}_\epsilon(w) = \min_{0 \leq g \leq w^\eta} -\mu^T (w^\xi + B^T g) + \kappa(\epsilon) \left\| \Sigma^{1/2} (w^\xi + B^T g) \right\|_2 \ldots \ldots - a^T g + e^T w^\eta
$$

- SOCP has better scalability properties than SDP.
WCPVaR minimization is equivalent to:

$$\min_{w \in \mathcal{W}} \max_{r \in \mathcal{U}_\epsilon^p} -w^T r.$$ 

where the uncertainty set $\mathcal{U}_\epsilon^p \subseteq \mathbb{R}^m$ is defined as

$$\mathcal{U}_\epsilon^p = \left\{ r \in \mathbb{R}^m : \exists \xi \in \mathbb{R}^n \text{ such that } (\xi - \mu)^T \Sigma^{-1} (\xi - \mu) \leq \kappa(\epsilon)^2 \text{ and } r = f(\xi) \right\}$$

Unlike $\mathcal{U}_\epsilon$, the set $\mathcal{U}_\epsilon^p$ is not symmetric!
Robust Optimization Perspective on WCPVaR

Zymler, Rustem and Kuhn

Worst-Case Value-at-Risk of Derivative Portfolios
Consider Black-Scholes Economy containing:

- Stocks A and B, a call on stock A, and a put on stock B.
- Stocks have drifts of 12% and 8%, and volatilities of 30% and 20%, with instantaneous correlation of 20%.
- Stocks are both $100.
- Options mature in 21 days and have strike prices $100.

Assume we hold equally weighted portfolio.

Goal: calculate VaR of portfolio in 21 days.

- Generate 5,000,000 end-of-period stock and option prices.
- Calculate first- and second-order moments from returns.
- Estimate VaR using: Monte-Carlo VaR, WCVaR, and WCPVaR.
Example: WCPVaR vs WCVaR

▶ At confidence level $\epsilon = 1\%$:
  ▶ WCVaR unrealistically high: 497%.
  ▶ WCVaR is 7 times larger than WCPVaR.
  ▶ WCPVaR is much closer to actual VaR.


- \(m - n\) derivatives can be exotic with arbitrary maturity time. Value of asset \(i = 1 \ldots m\) is representable as \(v_i(\tilde{\xi}, t)\).

- For short horizon time \(T\), second-order Taylor expansion is accurate approximation of \(\tilde{r}_i\):

\[
\tilde{r}_i = f_i(\tilde{\xi}) \approx \theta_i + \Delta_i^T \tilde{\xi} + \frac{1}{2} \tilde{\xi}^T \Gamma_i \tilde{\xi} \quad \forall i = 1, \ldots, m.
\]

- Portfolio return approximated by (possibly non-convex):

\[
\begin{align*}
\mathbf{w}^T \tilde{r} &= \mathbf{w}^T f(\xi) \\
&\approx \theta(\mathbf{w}) + \Delta(\mathbf{w})^T \tilde{\xi} + \frac{1}{2} \tilde{\xi}^T \Gamma(\mathbf{w}) \tilde{\xi},
\end{align*}
\]

where we use the auxiliary functions

\[
\begin{align*}
\theta(\mathbf{w}) &= \sum_{i=1}^{m} w_i \theta_i, \quad \Delta(\mathbf{w}) = \sum_{i=1}^{m} w_i \Delta_i, \quad \Gamma(\mathbf{w}) = \sum_{i=1}^{m} w_i \Gamma_i.
\end{align*}
\]

- We now allow short-sales of options in \(\mathbf{w}\)
Worst-Case Quadratic VaR (WCQVaR)

For any $w \in \mathcal{W}$, we define WCQVaR as

$$\min \left\{ \gamma : \max_{\mathcal{P} \in \mathcal{P}} \left\{ \gamma \leq -\theta(w) - \Delta(w)^T \tilde{\xi} - \frac{1}{2} \tilde{\xi}^T \Gamma(w) \tilde{\xi} \right\} \leq \epsilon \right\}$$

Theorem: SDP Reformulation of WCQVaR

WCQVaR can be found by solving an SDP:

$$\text{WCQVaR}_\epsilon(w) = \min \gamma$$

s. t. $M \in \mathbb{S}^{n+1}$, $\tau \in \mathbb{R}$, $\gamma \in \mathbb{R}$

$$\langle \Omega, M \rangle \leq \tau \epsilon, \quad M \succeq 0, \quad \tau \geq 0,$$

$$M + \begin{bmatrix} \Gamma(w) & \Delta(w) \\ \Delta(w)^T & -\tau + 2(\gamma + \theta(w)) \end{bmatrix} \succeq 0$$

▶ There seems to be no SOCP reformulation of WCQVaR.
WCQVaR minimization is equivalent to:

\[
\min_{w \in \mathcal{W}} \max_{Z \in \mathcal{U}_q^\epsilon} - \langle Q(w), Z \rangle
\]

where

\[
Q(w) = \begin{bmatrix}
\frac{1}{2} \Gamma(w) & \frac{1}{2} \Delta(w) \\
\frac{1}{2} \Delta(w)^T & \theta(w)
\end{bmatrix},
\]

and the uncertainty set \( \mathcal{U}_q^\epsilon \subseteq \mathbb{S}^{n+1} \) is defined as

\[
\mathcal{U}_q^\epsilon = \left\{ Z = \begin{bmatrix} X & \xi \\ \xi^T & 1 \end{bmatrix} \in \mathbb{S}^{n+1} : \Omega - \epsilon Z \succeq 0, \ Z \succeq 0 \right\}
\]

\( \mathcal{U}_q^\epsilon \) is lifted into \( \mathbb{S}^{n+1} \) to compensate for non-convexity.
There is a connection between $\mathcal{U}_\varepsilon \subseteq \mathbb{R}^m$ and $\mathcal{U}_\varepsilon^q \subseteq \mathbb{S}^{n+1}$.

If we impose: $\mathbf{w} \in \mathcal{W} \implies \mathbf{\Gamma}(\mathbf{w}) \succeq \mathbf{0}$ then robust optimization problem reduces to:

$$\min_{\mathbf{w} \in \mathcal{W}} \max_{\mathbf{r} \in \mathcal{U}_\varepsilon^q} -\mathbf{w}^T \mathbf{r}$$

where the uncertainty set $\mathcal{U}_\varepsilon^q \subseteq \mathbb{R}^m$ is defined as

$$\mathcal{U}_\varepsilon^q = \left\{ \mathbf{r} \in \mathbb{R}^m : \exists \mathbf{\xi} \in \mathbb{R}^n \text{ such that } (\mathbf{\xi} - \mathbf{\mu})^T \mathbf{\Sigma}^{-1}(\mathbf{\xi} - \mathbf{\mu}) \leq \kappa(\varepsilon)^2 \text{ and } r_i = \theta_i + \mathbf{\xi}^T \mathbf{\Delta}_i + \frac{1}{2} \mathbf{\xi}^T \mathbf{\Gamma}_i \mathbf{\xi} \ \forall i = 1, \ldots, m \right\}$$

Unlike $\mathcal{U}_\varepsilon$, the set $\mathcal{U}_\varepsilon^q$ is not symmetric!
Robust Optimization Perspective on WCQVaR

Zymler, Rustem and Kuhn

Worst-Case Value-at-Risk of Derivative Portfolios
Now we want to estimate VaR after 2 days (not 21 days).

- VaR not evaluated at option maturity times → use WCQVaR (not WCPVaR).
- Use Black-Scholes to calculate prices and greeks.

At $\epsilon = 1\%$: WCVaR still 3 times larger than WCQVaR.

Outperformance: option strat 56%, stock-only strat 12%.
Sharpe Ratio: option strat 0.97, stock-only strat 0.13.
Allocation option strategy: 89% stocks, 11% options.
Extensions

- Introduce additional risk-factors such as stochastic volatility.
- No new theory needed: delta-gamma-vega-vomma approximation is still quadratic.
- Copulas also model non-linear dependencies.
- They could be integrated into Worst-Case VaR framework.
Conclusions

- Worst-Case VaR makes \textit{weak assumptions} about probability distributions.
- Can be used to compute as well as optimize the risk of \textit{large scale} derivative portfolios.
- Options are risky, but their use in a robust optimization framework can offer substantial benefits.
- No quantitative risk method can completely replace sound investment analysis!
Questions?

- Paper available on optimization-online.org